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BRIEF COMMUNICATION

A NOTE ON THE AXISYMMETRIC INTERACTION OF PAIRS OF RISING, DEFORMING GAS BUBBLES

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INTRODUCTION

Many studies of the motion of gas bubbles are reported in the literature (Bhaga & Weber 1981; Clift *et al.* 1978; Harper 1972). Due to the complexity of the problem, much of the theory is confined to the steady motion of bubbles assumed to be spherical or oblate ellipsoidal (Moore 1963, 1965).

Nevertheless, the unsteady initial rise and deformation of bubbles has also received some attention. Most physically realistic models of gas bubble formation deal with a bubble emerging from some form of orifice (Longuet-Higgins *et al.* 1991; Oğuz & Prosperetti 1993). The rise from rest of an initially spherical gas bubble is a more fundamental problem and so also deserves study. The practical difficulties of experimentally investigating this were highlighted by Walters & Davidson (1962). Calculations of the unsteady rise of single deforming bubbles have been made by Lundgren & Mansour (1991) with the aim of examining vortex ring bubbles. In this short note we examine some of the effects on the motion and deformation due to the interaction of a pair of bubbles rising in an axisymmetric geometry.

RESULTS

In the calculations described below, we assume that the Reynolds number is sufficiently large that viscous effects can be neglected and we solve for the motion of the bubbles using a boundary integral method. A description of this technique can be found in, for example, Guerri *et al.* (1981) or in Best & Kucera (1992), a modified version of whose code was used for these calculations.

Before describing the numerical results for the interaction of a pair of gas bubbles, we shall summarize (see figure 1) the effect of bubble size on the rise of single bubbles (Lundgren & Mansour 1991). The extent of bubble deformation is characterized by the Eötvös number, given by $\mathrm{Eo} = 4\,\rho g a^2/\sigma$ where σ is surface tension and a is the bubble radius. For a large Eötvös number [Eo = 212, figure 1(a)], the effect of surface tension, which acts so as to prevent increases in surface area, is small and so a narrow jet of fluid is allowed to form below the bubble, eventually impacting on the upper surface. In reality, the bubble would then evolve into a toroidal vortex ring bubble (Lundgren & Mansour 1991). With decreasing Eötvös number, the jet becomes slower and less sharp. When the Eötvös number becomes less than about 32.3, the jet fails to reach the far side of the bubble before widening and forming a jet directed radially outwards, ultimately pinching off a toroidal bubble and leaving behind a spherical cap [see figure 1(b)]. As the Eötvös number decreases further, the volume of the toroidal bubble released decreases (see figure 2) owing to a yet shorter, wider jet. For the case Eo = 13.2, shown in figure 1(c), the bubble evolves into a shape resembling a skirted bubble. The jet does not impact on the side of the bubble but surface tension pulls the sharp rim upwards and towards the centre of the bubble. For smaller Eötvös number [Eo = 4.8, figure 1(d)] a very slow indentation forms at the rear, but it is too wide to result in the formation of a skirt.

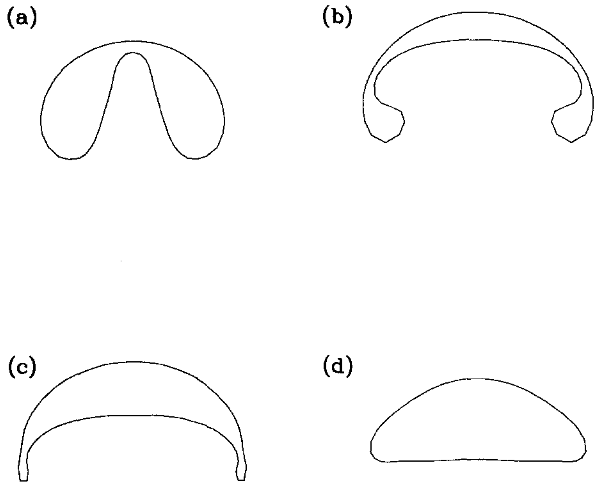

Figure 1. Shapes taken up by axisymmetric gas bubbles of various sizes. Smaller values of the Eötvös number correspond to smaller bubbles and thus a greater effect of surface tension. Eo takes the values (a) 212, (b) 29.8, (c) 13.2 and (d) 4.8, equivalent to air bubbles in water of radii 2 cm, 7.5, 5 and 3 mm.

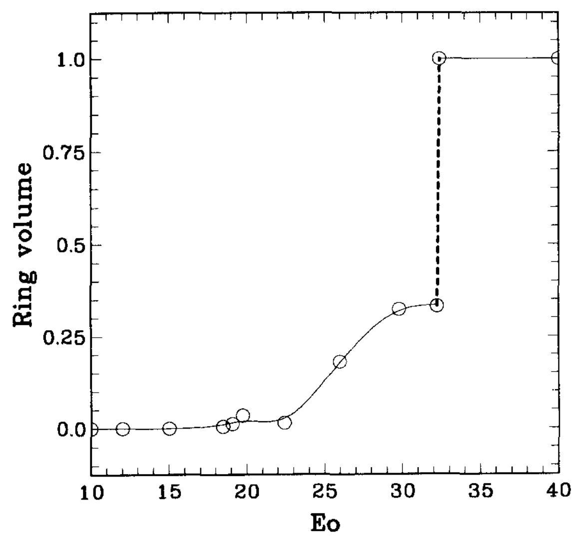

Figure 2. The non-dimensional volume fraction of the toroidal ring bubble formed when a single bubble becomes multiply-connected, plotted against the Eötvös number. Actual data points are indicated by a circle (○).

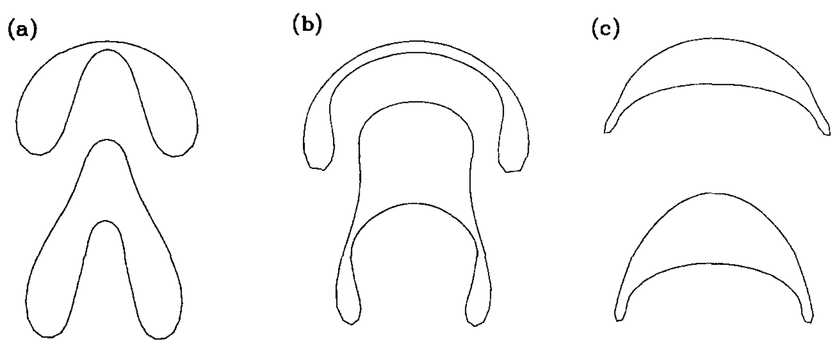

Figure 3. Calculated shapes of a pair of axisymmetric gas bubbles initially separated by 2.5 bubble radii. (a) Eo = 212, (b) Eo = 29.8 and (c) Eo = 13.2.

As mentioned briefly by Lundgren & Mansour (1991), one major difference between axisymmetric and two-dimensional bubble rise [see, for example, Boulton-Stone (1993)] is that for large axisymmetric bubbles [figure 1(a)] jets may penetrate the bubble and impinge on its upper surface, whereas in two-dimensions this is not so: even in the limit $Eo \rightarrow \infty$, the jet broadens out due to the downward pull of gravity before being able to reach the far side. Thus the broadening of the jet in the axisymmetric case [figure 1(b)] is a result of surface tension rather than of gravity.

There are a number of interesting effects brought about by the interaction of a rising bubble with a second nearby bubble. Here we examine a few cases of a pair of bubbles rising in an axisymmetric geometry, one bubble initially 2.5 bubble radii beneath the other.

Concentrating on those bubbles large enough for a jet to form (figure 3) we find the jet of the upper bubble to be noticeably broader than that of the lower bubble. In figure 3(a) the upper bubble

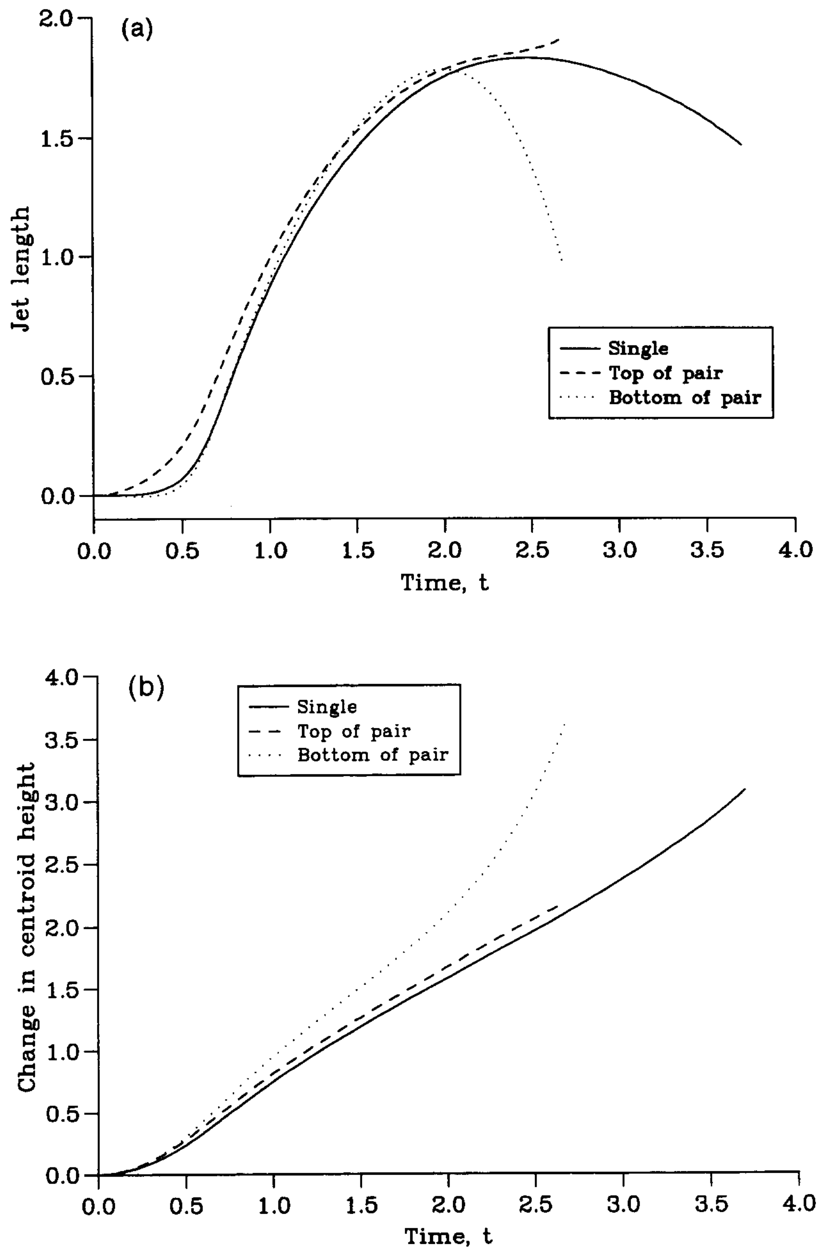

Figure 4. The effect of bubble–bubble interaction for the case Eo = 29.8 on: (a) the jet length approximated in terms of the height of the tip of the jet, Z_0, and the centroid height, Z_C, by $Z_0 - Z_C + 1$; and (b) the change in centroid height, $Z_C(t) - Z_C(0)$.

jet is also slightly broader than in the case of a single bubble of the same size [figure 1(a)]; comparison is more difficult for the smaller bubbles in figures 1(b) and 3(b) due to the earlier pinch-off of a toroidal bubble from the lower bubble, and hence break-down of the numerical code, when the second bubble is present.

A slightly increased pressure below the upper bubble, owing to the stagnation point at the top of the lower bubble, has the effect of making the tip speed of the jet on the upper bubble faster than for a single bubble of equal size. This is seen clearly in figure 4(a) for Eo = 29.8 (but the effect seems quite general), where the jet lengths, approximated by the change in distance between the jet tip and the bubble centroid, of the bubbles for the two-bubble case are compared against the single-bubble case. Further, for Eo = 29.8 (and this effect is much less pronounced for the larger bubbles), figure 4(a) shows the jet on the lower bubble to be faster than the jets of both the single bubble and of its companion bubble over much of the rise. This can be seen as a result of the lower bubble becoming thinner due to the flow around the upper bubble, therefore rising faster [see figure 4(b)]. A similar "slipstreaming" behaviour was reported for the two-dimensional case (Robinson *et al.* 1995).

Figure 4(a) indicates that, for Eo = 29.8, the length of the jet eventually decreases owing to the significant jet broadening which occurs for bubbles of this size. This is more noticeable for the lower bubble of the bubble pair due to the additional upward motion of the centroid induced through the draw-up of the top of the lower bubble into the jet of the upper bubble. This effect, seen in the bubble shapes of figure 3, is not as great as in the two-dimensional case (Robinson *et al.* 1995). We explain the difference in terms of the volume flow rate across a normal to a sphere, radius a, placed in a uniform stream with $\phi \sim Uz$ at infinity. We see that this is $U\pi a \sin^2\theta$, the unit length of normal. The corresponding rate for a cylinder is $Ul \sin\theta$, where l is the length of the cylinder. Since for small θ, the expression for the spherical case is an order of magnitude smaller, we may expect the fluid in the jet of the axisymmetric bubble to originate mainly from the sides rather than from directly underneath. Comparing Lundgren & Mansour's figure 2(a) to figure 1 from Boulton-Stone (1993)—both for large bubbles with buoyancy dominant—bears this out: the initial jet is noticeably narrower for the case of the cylindrical bubble. Consequently, in the axisymmetric case when a second bubble obstructs the flow beneath the upper bubble, the bulk of the volume of the jet comes from the sides, and so the draw-up of the lower bubble into the jet is not as great as in the two-dimensional case.

The reduced virtual mass of the bubble pair system, as compared with the single bubble, has the effect that the bubble pair, taken as a whole, rises faster. That this is the case is verified by reference to figure 4(b). The main contribution to the greater centroid velocity of a pair of bubbles is the more rapid rise of the lower bubble, as pointed out above; the upper bubble rises at a speed comparable to that of the single bubble.

For small Eötvös numbers, e.g. Eo = 13.2 shown in figure 3(c), the interaction between bubbles is less strong due to the widening of the bubble profiles which takes places as very broad jets form, thus effectively increasing bubble separation. In general, however, the interaction is as for the larger bubbles: the lower bubble becoming taller and narrower than the upper one.

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